

New approach to the superconducting qubit within a microcavity

Dagoberto S. Freitas^{1,2,†}, A. Vidiella-Barranco³, J. A. Roversi³

¹ Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, 30123-970, Belo Horizonte, MG, Brazil

² Departamento de Física, Universidade Estadual de Feira de Santana, 44036-900, Feira de Santana, BA, Brazil

³ Instituto de Física “Gleb Wataghin”, Universidade Estadual de Campinas, 13083-859 Campinas, SP, Brazil

PACS numbers: 42.50.Ct, 42.50.Pq, 74.50.+r

Abstract. We present a novel approach for studying the dynamics of a superconducting qubit in a cavity. We succeed in linearizing the Hamiltonian through the application of an appropriate unitary transformation followed by a rotating wave approximation (RWA). For certain values of the parameters involved, we show that it is possible to obtain specific hamiltonians. As an example, we show the existence of long time-scale revivals (super-revivals) for the excitation the qubit inversion.

1. Introduction

Superconducting (SC) Josephson junctions, which consist of a small superconducting electrode connected to a reservoir via a Josephson junction [1] are considered to be promising qubits for quantum information processing. Because of the charging effect in the small electrode, the two charge-number states, in which the number of Cooper pairs in the “box” electrode differs by one, constitute an effective two-level system. Superposition of the two charge states has been confirmed in experiments [2, 3], and coherent manipulation of the quantum state by using a dc gate-voltage pulse has been demonstrated; i.e., coherent oscillations between two degenerate charge states have been observed [4]. These “artificial atoms,” with well-defined discrete energy levels, provide a platform to test fundamental quantum effects, e.g., related to cavity quantum electrodynamics (cavity QED). The study of the cavity QED-type-system such as a SC qubit (see reference [5]) can also open new directions for studying the interaction between light and solid-state quantum devices. That may result in novel controllable electro-optical quantum devices in the microwave regime, such as microwave

† E-mail: dfreitas@uefs.br

Permanent address: Departamento de Física, Universidade Estadual de Feira de Santana, 44036-900, Feira de Santana, BA, Brazil.

single-photon generators and detectors. Cavity QED could also allow the transfer of information among SC qubits via photons, that may be used as information buses. Several theoretical proposals have analyzed the interaction between SC qubits with either quantized [5, 6, 7, 8, 9, 10, 11] or classical fields [12, 13, 14]. However, we believe that more attention should be paid to SC circuits embedded inside a QED cavity given the versatility provided by its coupling to photons. In this paper we make use of the same model as presented in [8], a single mode cavity field in the microwave regime, with photon transitions between the ground and first excited states of a macroscopic two-level system formed by a superconducting quantum interference device (SQUID). This artificial two-level “atom” can be easily controlled by an applied gate voltage V_g and the flux Φ_c generated by the classical magnetic field through the SQUID (e.g., [1, 5]), a process similar to that of a micromaser [16].

Here we adopt a new approach to this problem: we depart from the full radiation field and superconductor interaction Hamiltonian, and apply a unitary transformation similar to the one introduced in [15]. After transforming the original Hamiltonian, we show that it is possible to obtain simpler hamiltonians under certain resonance regimes. In particular, we show that after performing just a rotating wave approximation (RWA), we are able to obtain a Jaynes-Cummings like Hamiltonian and solve the problem analytically without making any further approximations.

2. The model

We consider a system constituted by a SQUID type superconducting box with n_c excess Cooper-pair charges connected to a superconducting loop via two identical Josephson junctions having capacitors C_J and coupling energies E_J . An external control voltage V_g couples to the box via a capacitor C_g . We also assume that the system operates in a regime, consistent with most experiments involving charge qubits, in which only Cooper pairs coherently tunnel in the superconducting junctions. Therefore the system Hamiltonian may be written as [1]

$$H_{qb} = 4E_{ch}(n_c - n_g)^2 - 2E_J \cos\left(\frac{\pi\Phi_X}{\Phi_0}\right) \cos(\Theta), \quad (1)$$

where $E_{ch} = e^2/2(C_g + 2C_J)$ is the single-electron charging energy, $n_g = C_g V_g/2e$ is the dimensionless gate charge (controlled by V_g), Φ_X is the total flux through the SQUID loop and Φ_0 the flux quantum. By adjusting the flux through the superconducting loop, one may control the Josephson coupling energy as well as switch on and off the qubit-field interaction. The phase $\Theta = (\phi_1 + \phi_2)/2$ is the quantum-mechanical conjugate of the number operator n_c of the Cooper pairs in the box, where ϕ_i ($i = 1, 2$) is the phase difference for each junction. The superconducting box is assumed to be working in the charging regime and the superconducting energy gap Δ is considered to be the largest energy involved. Moreover, the temperature T is low enough so that the conditions $\Delta \gg E_{ch} \gg E_J \gg k_B T$ hold. The superconducting box then becomes a two-level system with states $|g\rangle$ (for $n_c = 0$) and $|e\rangle$ (for $n_c = 1$) given that the gate voltage is

near a degeneracy point ($n_g = 1/2$) [1] and the quasi-particle excitation is completely suppressed [17].

If the circuit is placed within a single-mode microwave superconducting cavity, the qubit can be coupled to both a classical magnetic field (generates a flux Φ_c) and the quantized cavity field (generates a flux $\Phi_q = \eta a + \eta^* a^\dagger$, with a and a^\dagger the annihilation and creation operators), being the total flux through the SQUID $\Phi_X = \Phi_c + \Phi_q$ [5, 6, 7]. The parameter η is related to the mode function of the cavity field. The system Hamiltonian will then read

$$H = \hbar\omega a^\dagger a + E_z \sigma_z - E_J(\sigma_+ + \sigma_-) \cos(\gamma I + \beta a + \beta^* a^\dagger), \quad (2)$$

where we have defined the parameters $\gamma = \pi\Phi_c/\Phi_0$ and $\beta = \pi\eta/\Phi_0$. The first term corresponds to the free cavity field with frequency $\omega = 4E_{ch}/\hbar$ and the second one to the qubit having energy $E_z = -2E_{ch}(1 - 2n_g)$ with $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$. The third term is the (nonlinear) photon-qubit interaction term which may be controlled by the classical flux Φ_c . In general the Hamiltonian in equation (2) is linearized under some kind of assumption. In [8], for instance, the authors decomposed the cosine in Eq.(2) and expanded the terms $\sin[\pi(\eta a + H.c.)/\Phi_0]$ and $\cos[\pi(\eta a + H.c.)/\Phi_0]$ as power series in a (a^\dagger). In this way, if the limit $|\beta| \ll 1$ is taken, only single-photon transition terms in the expansion are kept, and a Jaynes-Cummings type Hamiltonian (JCM) is then obtained. Here, in contrast to that, we adopt a similar technique to the one presented in reference [15] obtaining a linear (JCM) Hamiltonian valid for any value of $|\beta|$. We first apply a unitary transformation to the full Hamiltonian and make approximations afterwards. As we are going to see, this will allow us to explore new regimes in that physical system. First we write equation (2) in the interaction representation, $H_I = U_0^\dagger V U_0$ with

$$U_0 = \exp[-i(\omega a^\dagger a + \frac{E_z}{\hbar} \sigma_z)t] \quad (3)$$

and

$$V = -E_J(\sigma_+ + \sigma_-) \cos(\gamma I + \beta a + \beta^* a^\dagger), \quad (4)$$

obtaining

$$\begin{aligned} H_I = & -\frac{E_J}{2} \{ \sigma_+ e^{2i\frac{E_z}{\hbar}t} \exp[i(\beta a e^{-i\omega t} \\ & + \beta^* a^\dagger e^{i\omega t} + \gamma)] + h.c. \} \\ & -\frac{E_J}{2} \{ \sigma_- e^{2i\frac{E_z}{\hbar}t} \exp[-i(\beta a e^{-i\omega t} \\ & + \beta^* a^\dagger e^{i\omega t} + \gamma)] + h.c. \}. \end{aligned} \quad (5)$$

By applying the unitary transformation

$$\begin{aligned} T = & \frac{1}{\sqrt{2}} \left\{ -\frac{1}{2} [D^\dagger(\xi, \gamma) - D(\xi, \gamma)] I \right. \\ & -\frac{1}{2} [D^\dagger(\xi, \gamma) + D(\xi, \gamma)] \sigma_z \\ & \left. + D(\xi, \gamma) \sigma_+ + D^\dagger(\xi, \gamma) \sigma_- \right\} \end{aligned} \quad (6)$$

to the Hamiltonian in equation (5), where $D(\xi, \gamma) = e^{i\frac{E_J}{\hbar}t} \exp[\frac{1}{2}(\xi(t)a^\dagger - \xi^*(t)a)]e^{i\frac{\gamma}{2}}$ is Glauber's displacement operator, with $\xi(t) = i\beta^*e^{i\omega t}$, we obtain the following transformed Hamiltonian

$$\begin{aligned}
H_T &\equiv TH_I T^\dagger \\
&= \frac{E_J}{2}\sigma_z + i\frac{\hbar}{2}[\omega(\beta a e^{-i\omega t} - \beta^* a^\dagger e^{i\omega t}) + 2i\frac{E_J}{\hbar}] \\
&\quad \times (\sigma_+ + \sigma_-) \\
&\quad + \frac{E_J}{2}\cos[2(\beta a e^{-i\omega t} + \beta^* a^\dagger e^{i\omega t}) + 2\gamma]\sigma_z \\
&\quad - i\frac{E_J}{2}\sin[2(\beta a e^{-i\omega t} + \beta^* a^\dagger e^{i\omega t}) + 2\gamma] \\
&\quad \times (\sigma_+ - \sigma_-).
\end{aligned} \tag{7}$$

In order to eliminate the temporal dependence in the transformed Hamiltonian H_T we rewrite it in an interaction representation in the transformed space, or $H_{TI} = U_{0T}^\dagger V_T U_{0T}$, where

$$\begin{aligned}
V_T &= i\frac{\hbar}{2}[\omega(\beta a e^{-i\omega t} - \beta^* a^\dagger e^{i\omega t}) + 2i\frac{E_J}{\hbar}](\sigma_+ + \sigma_-) \\
&\quad + \frac{E_J}{2}\cos[2(\beta a e^{-i\omega t} + \beta^* a^\dagger e^{i\omega t}) + 2\gamma]\sigma_z \\
&\quad - i\frac{E_J}{2}\sin[2(\beta a e^{-i\omega t} + \beta^* a^\dagger e^{i\omega t}) + 2\gamma](\sigma_+ - \sigma_-)
\end{aligned} \tag{8}$$

and $U_{0T} = e^{-i\frac{E_J}{2\hbar}\sigma_z t}$. We then obtain the transformed Hamiltonian in the interaction representation

$$\begin{aligned}
H_{TI} &= \frac{i\hbar\omega}{2} \left[\left(\beta a \sigma_+ e^{-i(\omega - E_J/\hbar)t} - \beta^* a^\dagger \sigma_- e^{i(\omega - E_J/\hbar)t} \right) \right. \\
&\quad \left. - \left(\beta^* a^\dagger \sigma_+ e^{i(\omega + E_J/\hbar)t} - \beta a \sigma_- e^{-i(\omega + E_J/\hbar)t} \right) \right] \\
&\quad - \hbar\omega_0 \left(\sigma_+ e^{E_J/\hbar t} + \sigma_- e^{-E_J/\hbar t} \right) \\
&\quad + \frac{E_J}{2} \cos \left[2 \left(\beta^* a^\dagger e^{i\omega t} + \beta a e^{-i\omega t} \right) + 2\gamma \right] \sigma_z \\
&\quad - i\frac{E_J}{2} \sum_{n=0}^{\infty} \frac{(2i)^{2n}}{(2n)!} \left\{ \left\{ \sin(2\gamma) e^{i\frac{E_J}{\hbar}t} + \frac{2\cos(2\gamma)}{(2n+1)} \right. \right. \\
&\quad \times \left[\beta^* a^\dagger e^{i(\omega + E_J/\hbar)t} + \beta a e^{-i(\omega - E_J/\hbar)t} \right] \left. \right\} \\
&\quad \times \left[\beta^{*2} a^{\dagger 2} e^{2i\omega t} + \beta^2 a^2 e^{-2i\omega t} \right. \\
&\quad \left. \left. + |\beta|^2 (a a^\dagger + a^\dagger a) \right]^n \sigma_+ - h.c. \right\}.
\end{aligned} \tag{9}$$

3. Resonance conditions

At this stage we have in hands a transformed Hamiltonian equation (9) with a more complicated structure than the original one, in equation (2). Nevertheless, our new Hamiltonian may be considerably simplified by choosing appropriate resonance conditions and applying the RWA. In fact, as we are going to show below, for certain values of the cavity frequency ω and the parameter γ , it is possible to (approximately) obtain well known Hamiltonians of quantum optical resonance, such as the one-photon and two-photon Jaynes-Cummings Hamiltonians. We would like to remark that these are Hamiltonians in the transformed space, though.

3.1. Case I: One-photon process

If the cavity frequency is such that $\hbar\omega = E_J$ and the parameter γ (which may be controlled by the classical flux) equals $\gamma = \pi/4$ rad, the rapidly oscillating terms in the right hand side of equation (9) may be neglected (RWA), and the transformed Hamiltonian above reduces to the one-photon Hamiltonian

$$H_{TI,1} \approx -i\hbar(g^*a^\dagger\sigma_- - ga\sigma_+), \quad (10)$$

which coincides with the Jaynes-Cummings Hamiltonian with effective coupling constant $|g| = |\beta|\omega/2$. We should remark that this result holds for any value of $|\beta|$, in contrast to the approach in reference [8], where all higher order powers of $|\beta|$ were neglected and only a single-photon transition term is kept in the expansion of eq. (2), which leads to a Hamiltonian valid only in the limit of $|\beta| \ll 1$.

Now we would like to discuss the dynamics of the system in the one-photon process. We should remark that despite of the fact that we have obtained a Jaynes-Cummings type Hamiltonian in the transformed space, the system dynamics is closely related to the dynamics of the driven Jaynes-Cummings model (DJCM), instead. In this case, the time evolution of the state vector, for an initial state $|\psi(0)\rangle$ is

$$|\psi(t)\rangle = T^\dagger(t)U_{0T}(t)U_I(t)T(0)|\psi(0)\rangle, \quad (11)$$

where $U_I(t)$ is the Jaynes-Cummings evolution operator [18] in the interaction representation

$$\begin{aligned} U_I(t) = & \frac{1}{2}[C_{n+1} + C_n]I + \frac{1}{2}[C_{n+1} - C_n]\sigma_z \\ & + \frac{\beta}{|\beta|}S_{n+1}a\sigma_+ - \frac{\beta^*}{|\beta|}a^\dagger S_{n+1}\sigma_-, \end{aligned} \quad (12)$$

with $C_{n+1} = \cos(|g|t\sqrt{aa^\dagger})$ and $S_{n+1} = \sin(|g|t\sqrt{aa^\dagger})/\sqrt{aa^\dagger}$.

For an initial state $|\psi(0)\rangle = |e\rangle|\alpha\rangle$, or the qubit in the excited state and the field in a coherent state, the qubit inversion or $W(t) = \langle\sigma_z\rangle$ is given by,

$$W(t) = \frac{1}{2} \exp(-|\alpha - \beta|^2) \sum_{n=0}^{\infty} \frac{|\alpha - \beta|^{2n}}{n!} \left\{ \right.$$

$$\begin{aligned}
& 2 \cos\left(\frac{E_J}{\hbar}t\right) c_{n+1} c_n + \frac{n!}{\sqrt{n!(n+1)!}} \\
& \times \left[(\alpha^* - \beta^*) e^{i\frac{E_J}{\hbar}t} + (\alpha - \beta) e^{-i\frac{E_J}{\hbar}t} \right] \\
& \times (c_{n+2} - c_n) s_{n+1} - \frac{n!}{\sqrt{n!(n+2)!}} \left[\right. \\
& \left. (\alpha^* - \beta^*)^2 e^{i\frac{E_J}{\hbar}t} + (\alpha - \beta)^2 e^{-i\frac{E_J}{\hbar}t} \right] \\
& \times s_{n+2} s_{n+1} \Big\}, \tag{13}
\end{aligned}$$

where $c_n = \cos(gt\sqrt{n})$ and $s_n = \sin(gt\sqrt{n})$. In equation (13) the parameter β was considered real for simplicity.

The structure of the equation above is similar to the one obtained for $\langle \sigma_z \rangle$ in the DJCM [19]. Therefore we expect the phenomenon of super-revivals to be present in the qubit-cavity system, analogously to the DJCM. Super-revivals are long time scale revivals arising in the atom-field dynamics. This peculiar behavior is illustrated in figure (1) and figure (2). We note that the existence or not of super-revivals is narrowly connected to the preparation of the initial field state. For instance, if we have $\alpha = (5.0, 0.5)$, the super-revivals do not occur [see Fig. (1)], and we have ordinary revivals only. However, for appropriate values of the parameters, for instance, $\alpha = (0.5, 5.0)$, super-revivals take place in that system [see Fig. (2)].

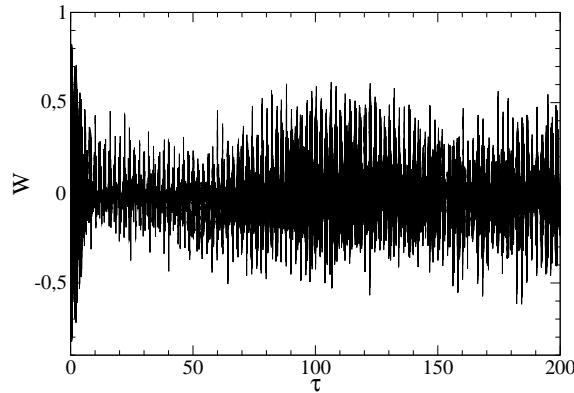


Figure 1. Plot of the qubit inversion as a function of the (dimensionless) scaled time $\tau = gt$. Ordinary revivals occurring for $\alpha = (5.0, 0.5)$ and $\beta = 0.2$.

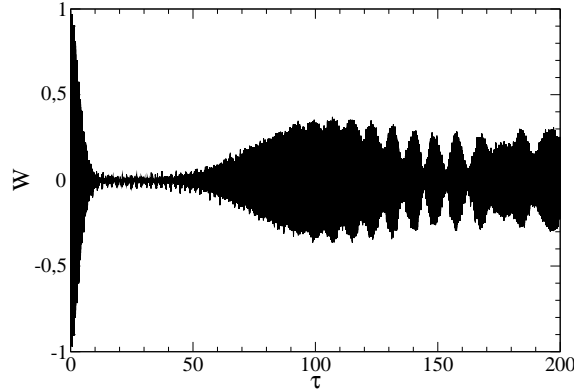


Figure 2. Plot of the qubit inversion as a function of the (dimensionless) scaled time $\tau = gt$. Super-revivals occurring for $\alpha = (0.5, 5.0)$ and $\beta = 0.2$.

3.2. Case II: Multi-photon processes

If we choose different resonance conditions in Hamiltonian (9), such as, for instance, $2\hbar\omega = E_J$ and keep $\gamma = \pi/4$ rad, the dominant terms will be second order terms in $|\beta|$. If in addition to that the relation $|\beta|^4 \ll 1$ is satisfied, the resulting Hamiltonian is

$$H_{TI,2} \approx i \frac{E_J}{2} \left[\beta^{*2} a^{\dagger 2} \sigma_+ - \beta^2 a^2 \sigma_+ \right], \quad (14)$$

which is a typical Hamiltonian describing two-photon processes. Higher order multi-photon processes may also be obtained, but in addition to the resonance condition it is now necessary to have $|\beta|$ small.

4. Conclusion

In conclusion, we have presented a novel approach for studying the dynamics of a superconducting qubit interacting with the quantized field within a highly reflective cavity. In general, approximations are made directly to the full Hamiltonian in equation (2). In our method, we first apply an unitary transformation to the Hamiltonian (2) and make the relevant approximations after performing the transformation. This constitutes the main result of our paper. For instance, if a specific resonance condition ($\hbar\omega = E_J$) is chosen (as well as $\gamma = \pi/4$ rad) we obtain a Jaynes-Cummings-type Hamiltonian after applying the RWA. We remark that this was done without the need of taking the limit of $|\beta| \ll 1$, in contrast to other approaches. As a consequence, in that regime of resonance, the superconducting system may exhibit typical behavior of a driven Jaynes-Cummings model [19] (or a trapped ion within a cavity [15]). In particular, we have predicted the existence of long time-scale revivals (super-revivals) for the excitation of the superconducting qubit inversion. Our method may be further explored: if we choose a different resonance condition, e.g., $2\hbar\omega = E_J$, two photon transitions will be

the dominant processes, provided that $|\beta|^4 \ll 1$. Our approach allows us not only to establish a direct connection to other well known models in quantum optics, but also the exploration of different regimes in superconducting systems.

Acknowledgments

D.S.F thanks the financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico-CNPq (150232/2012-8), Brazil. We thank FAPESP and CNPq for financial support through the National Institute for Science and Technology of Quantum Information (INCT-IQ) and the Optics and Photonics Research Center (CePOF).

References

- [1] Yu. Makhlin, G. Schn, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
- [2] V. Bouchiat, D. Vion, P. Joyez, D. Esteve, and M. H. Devoret, Phys. Scr. T **76**, 165 (1998).
- [3] Y. Nakamura, C. D. Chen, and J. S. Tsai, Phys. Rev. Lett. **79**, 2328 (1997).
- [4] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
- [5] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).
- [6] W. A. Al-Saidi and D. Stroud, Phys. Rev. B **65**, 224512 (2002).
- [7] Shi-Liang Zhu, Z. D. Wang and Kaiyu Yang, Phys. Rev. A **68**, 034303 (2003).
- [8] Yu-xi Liu, L. F. Wei, and F. Nori, Europhys. Lett. **67**, 941 (2004); Yu-xi Liu, L. F. Wei, and F. Nori, Phys. Rev. **A** **71**, 063820 (2005).
- [9] Y. B. Gao, Y. D. Wang, and C. P. Sun, Phys. Rev. **A** **71**, 032302 (2005).
- [10] C. P. Yang, Shih-I Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).
- [11] A. M. Zagorskin, M. Grajcar, and A. N. Omelyanchouk, Phys. Rev. A **70**, 060301(R) (2004).
- [12] Z. Zhou, Shih-I Chu, and S. Han, Phys. Rev. **B** **66**, 054527 (2002).
- [13] E. Paspalakis and N. J. Kylstra, J. Mod. Opt. **51**, 1679 (2004).
- [14] Yu-xi Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, Phys. Rev. Lett. **95**, 087001 (2005).
- [15] H. Moya-Cessa, A. Vidiella-Barranco, J.A. Roversi, D.S. Freitas and S.M. Dutra, Phys. Rev. A **59**, 2518 (1999).
- [16] J. Krause, M.O. Scully, H. Walther, Phys. Rev. A **36**, 4547 (1987).
- [17] D. V. Averin, and Y. V. Nazarov, Phys. Rev. Lett. **69**, 1993 (1992).
- [18] S. Stenholm, Phys. Rep. C **6**, 1 (1973).
- [19] S. M. Dutra, P. L. Knight, and H. Moya-Cessa, Phys. Rev. A **49**, 1993 (1994).